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Washington Project

Division 5, National Defense Research Committee
of the
Office of Scientific Research and Development

Some Aspects of the Design of Homing Aero-Missiles

by

Hugh L. Dryden
National Bureau of Standards

A report to Division 5
from the
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Approved for National Bureau of Standards by

Lyman J. Briggs, Director.

Preface

The work described in this report is pertinent to the projects designated by the War Department Liaison officer as AC-1, AC-36, and AC-42 and to the projects designated by the Navy Department Liaison officer as NO-115, NO-174, and NO-235. This work was carried out and reported by National Bureau of Standards under a transfer of funds from OSRD with the co-operation of the Washington Radar Group of the Massachusetts Institute of Technology and Section Re4g of the Bureau of Ordnance, Navy Department.

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I. Introduction

Modern warfare is primarily a warfare of missiles - bullets, shells, rockets, bombs, and grenades. The chief problems are to transport, project or propel, and direct these missiles accurately to strike targets of military importance. Missiles may be transported to the vicinity of a target by men on foot, by mobile artillery, by tanks, by aircraft, or by ships. They may be dropped from aircraft, projected from guns on land, ship, or aircraft, accelerated for all or parts of their trajectories by rocket motors, or they may be self-propelled. Most of the missiles in current use are not under the control of the user after launching. Some form of sighting device is necessary to establish the initial direction of travel in a manner to cause the missile to strike the target. Gun sights, bomb sights, and the complicated gun directors and computers are mere aids to determine where to point the gun or release the bomb, and after release of the missile no further control is possible.

For many years much thought has been given to the possibilities of guiding missiles after their release to correct for errors in sighting and for evasive action of moving targets. The development of radio communication stimulated inventive activity in this field and even during the first World War there were experiments on radio-directed aircraft. There are innumerable patent applications relating not only to radio-

control but also to other aspects of guided missiles. Particu-
larly attractive has been the idea of missiles which will seek
out a target automatically or "home" on the target.

During the present war much serious scientific effort has
been devoted to a full exploration of the practicability of
guided missiles as useful weapons. The German experiments
with HS 293 and FX and later with V-1 and V-2 have stimulated
the interest of our military leaders and enlisted their support
in the fullest exploitation of the possibilities of this type
of weapon.

Classification of Missiles.

We may recognize five possible classes of missile from
the standpoint of the source of control. The first, illustrated
by the German V-1 and V-2, uses an autopilot to hold the missile
on a prescribed course. The missile receives no information
relating to the target, and must be launched in the direction
required to strike the target. This type is really a kind of
long range artillery although the missile itself carries a part
of the computer.

The second type, illustrated by the Japanese suicide bombers,
carries a human pilot in the missile, who is expended with the
missile. This solution is known, involves no new technical
problems, but is not attractive to a nation placing a high
value on human life.

The third type, illustrated by the German HS 293 and FX and the Division 5 NDRC Azon and Razon series, uses a remote human pilot who may either see the missile and target visually, aided by flares on the missile and by sighting devices, or be guided by a repeat-back of television or radar information from apparatus contained in the missile, or be guided by radar location of the missile in relation to map location of the target or radar location of the target.

The fourth type, of which there is no existing illustration, uses some type of beam directed toward the target which the missile automatically follows.

The fifth type is the target-seeking or homing type, illustrated by Division 5 NDRC Pelican, Bat, and Felix. Such a missile must utilize some physical property of the military target which causes it to stand out from the background. The most commonly suggested property is the emission or reflection of electromagnetic radiation. Separate techniques are available for transforming three major divisions of the electromagnetic spectrum into directional information. Of the three - visible light, infra red, and radio - radio frequencies, for technical reasons, hold the most promise of useful weapons. Radio techniques as developed in radar are directly applicable. Visible light and infra-red are useful for certain specific types of targets. Other properties have been suggested, for example, the emission or reflection of sound. Investigations along

this line for aero-missiles have not given promising results.

Necessity of Technical Coordination.

This report is concerned with various aspects of the design of this last type, the homing aero-missile. The impression is prevalent that scientific advances in many fields have progressed to the point where the development of such a missile is purely a matter of engineering design on the part of the several specialist groups with the usual coordination as to dimensional requirements, weights, and time of completion. Experience has taught otherwise. Optimistic time schedules based on such an assumption can not be met. The development of successful homing aero-missiles requires the solution of certain research problems associated with the complete article involving complex relationships between the performance characteristics of the component parts. There is required a type of overall technical coordination beyond that required in the design of aircraft as ordinarily practiced.

The required technical coordination is made difficult by the wide variety of specialists of different scientific and technical background and accustomed to different vocabularies and habits of thought whose work must be coordinated. For example, in the case of a propelled radar homing missile there will be represented experts in aerodynamics, aircraft structures, propulsion, servomechanisms, electronics, radar, computers, explosives, and fuses. Other types of missiles will require

experts in radio, optics, infra-red radiation, heat, etc. No
one person can be expert in all of these diverse fields, but
the success of the project requires a project engineer who has
sufficient knowledge of these fields to be able with the help
of advice from the specialists to assume technical leadership
in the solution of research problems associated with the system
as a whole.

In the design of any homing missile, there soon emerge a
number of problems which cut across the boundaries of the
specialist groups. The particular design which seems best to
one group of specialists creates difficult problems for other
groups, and the requirements put forth by the several groups
as optimum are often contradictory. For example, certain errors
are introduced unless the intelligence device "looks" along
the direction of motion, i.e. is accurately bore-sighted. The
conventional airplane flies at an angle of attack which varies
with the position of the elevator. If the aerodynamics
specialist adopts a conventional aircraft design with elevator
control, the intelligence device must be coupled to the elevator
control in such a manner as to compensate for variations of
angle of attack for all conditions of flight. However, the
designer of the intelligence device might properly suggest that
the aerodynamicist design a vehicle which does not change its
attitude with application of the controls. It becomes a matter
of research to determine which solution gives greater accuracy

and hence how responsibility is to be allocated between the two
specialists.

Purpose and Background of this Report.

These broad aspects of the design of homing aero-missiles
are treated in the present report. An attempt has been made to
make the discussion general in character and applicable to all
such missiles, whether propelled or not.   It should be stated,
however, that the discussion arises from experience with the
radar homing missiles of the Pelican and Bat series, and this
account undoubtedly reflects the solutions there adopted, as
well as the problems peculiar to radar homing.

II. Target Discrimination and Tracking
Limitations of Mechanisms as Compared to Human Brain.

In visual shooting or bombing, the target is identified by
the pattern of optical radiation as perceived by the human eye
and interpreted by the human brain. In radar fire control or
bombing, the target is identified by the pattern of short-wave
electromagnetic radiation exhibited on the screen of a cathode
ray tube as perceived by the human eye and interpreted by the
human brain. During the flight of a homing aero-missile these
radiation patterns must be made to operate control mechanisms
and the element of interpretation by a human brain is absent.
This introduces many problems and severe limitations. A
mechanism can perceive only a limited number of physical charac-
teristics of the target pattern and can exercise no judgment

in interpreting the received information except of the simplest
kind, for example, smoothing over a certain time interval. Thus
at present no mechanism is known which will select a given
target from an optical image of a complex visual pattern. In
their present state of development homing missiles can utilize
only simple target situations, the most favorable target
situation being that of a small number of ships on a large body
of water, or more generally, a few isolated discontinuities
in a radiation pattern which is otherwise nearly uniform. From
this point of view, aircraft also present favorable target
situations.

Principles of Target Discrimination.

Excluding the case of a single isolated discontinuity in
the radiation pattern, which seldom occurs, the first main
problem of any homing missile is to determine the method or
methods to be used for target discrimination, when several
targets are present. In the simpler devices the missile homes
on the biggest discontinuity within its field of view. In
many homing missiles, means are provided to permit the initial
selection of target by a human operator who often has available
information supplemental to that given to the control circuits
of the missile. Means may also be provided within the missile
to keep track of the initially selected target and to respond
only to signals from that target. Even in the simpler missiles,
the operator attempts to release the missile either with a

single lerge disoontinuity within the field or on such a peth
that a single target comes within the field end depends on the
preoision of the oontrols of the missile to keep the miesile
traoking this target.

In order to obtain informetion about the looation of the
target, a homing miesile must be directionally sensitive.
Usually the received intensity of rediation ie greateet when
the axis of the rediation-receiver is pointed direotly at the
target. Thus Fig. 1 shows a portion of the response ourve of
the reoeiving antenna of the radar reoeiver ueed in the Peliocan
project. The relative power ia plotted in terme of decibels,
a logarithmio unit, but the power ratioe are also indioated.
The width at 1/2 power ia 23° and the width at 1/10th power ia
43°. In other words e terget et bearing 21 1/2° from the
antenna axis gives one-tenth as muoh energy to the reoeiver as
a target on the axis giving the same intensity of radiation.

The radietion pattern shown oen not be used directly
because there is no discrimination between right and left or up
and down. The usual praotice ie to soan the field of view, to
oommutete or phase the reoeived signal intensity with the
soanning, end compare right with left and up with down or per-
form comparisons in some other ooordinate system. The on-course
indioation becomes then an equelity of two signals and a
directionel sense is provided. In Pelioan, oonioal soanning
is ueed, the antenna axis desoribing a oone of 11° half-vertex
angle. A commutetor provides the phasing. If there were a

variety of targets of equal intensity, every one within a cone of half-vertex angle of approximately 22° would give signals of one-half maximum power or more. This represents one method of stating a figure for the field of view of the receiver, all targets being assumed to return signals of equal intensity.

In Pelican, the target is "illuminated" by a radar beam, and the directional characteristics of the transmitter antenna provide additional directional discrimination, which is not of interest in this discussion. In Bat, the missile carries the transmitter and the same antenna is used for transmission and reception. The field of view is accordingly smaller than for Pelican.

In other types of intelligence devices, much smaller fields of view are used, the width at 1/2 power being 10° or less. This is advantageous from the point of view of directional discrimination of targets. Limitation of the field of view is the first general method of securing target discrimination. However, a narrow field of view introduces tracking problems as will be discussed later.

The second method of securing target discrimination is by means of signal strength. This can not be entirely separated from the directional properties of the intelligence system, and permits little choice other than to home on the strongest signal. It is usually necessary to include some type of automatic gain control to obtain directional information at signal levels which

may vary more than a billion fold as the missile approaches the target. The strongest signal within the field of view will govern the sensitivity of the receiver through the action of the automatic gain control.

Target discrimination may also be secured through the selective action of the intelligence device in responding to radiation within narrow wave length limits. This is best utilized when an intense beam of the desired radiation can be concentrated on the target and the missile made sensitive only to wave lengths within the narrow limits of the transmitted radiation.

In systems in which a pulsed illuminating beam is used, as exemplified particularly in the Pelican and Bat radar homing systems, another method of target discrimination may be used. This is range selection and synchronization. By making the intelligence system sensitive only for a short period at a predetermined time following the emission of an illuminating pulse, the control information can be restricted to that received from targets lying within certain range limits, say within a zone of ±250 feet of the actual target range. For a given range corresponds to a definite time of transit of the pulse from transmitter to target to receiver. Naturally such a range selecter requires an automatic method of tracking the target in range as the range decreases. The operation of the range selecter also requires the synchronization of the receiver and

the transmitter. The synchronization is effective in dis-
criminating against reflected radiation originating from other
transmitters operating on the same radio frequency but with
different pulse rates.

The use of range selection is found to be essential in
radar homing missiles launched from aircraft because of the
so-called "altitude signal", i.e. energy returned from the
earth directly below the aircraft. If a reasonable cone of
vision is to be maintained, the directional selectivity of the
antenna is inadequate to discriminate against the large
reflecting area lying beneath the aircraft. A range selector
and range tracking device makes possible the elimination of
this signal in the case of glider missiles, since the altitude
is always less than the range to the target. Presently avail-
able radar homing devices can not be used under conditions
where the target may appear at the same range as the altitude
signal, for example in air to air missiles at ranges greater
than the altitude. In the case of ground to air missiles
using receiver only, the geometry is more favorable than for
the glider, and the range of the target will not coincide with
that of the altitude signal. For a send-receive missile, the
missile will at some time be at the same altitude as the range
to the target and hence may thereafter home on the altitude
signal.

It is possible to devise more complex mechanisms to perform

more difficult feets of discrimination, for example, to permit
the launching of a radar homing missile at long range without
advance selection of a target and to have the range selection
device search for, choose, and look on s target when the missile
has proceeded a definite distance. The limit of performance
is set only by the permissible complexity of the mechanism.

The suggestion has often been made that lower animals be
used as intelligence devices, since their brain, like that of
the human one, can perform difficult tasks of discrimination.
This possibility is perhaps the only one of adequate complexity
to deal with the pattern discrimination required to select, for
example, a particular building within the complex optical
radiation pattern presented by a city. Proponents of this
method point out that near mechanical reliability may be realized
in animals by establishing in them a conditioned reflex
associated with the object selected for attention.

Relation between Field of View and Permissible Motion of Vehicle.

After a target has been selected by the operator before
release of the missile or by the mechanism of the missile itself,
the target must be tracked, i.e. held within the field of view
of the missile during the remainder of its flight. The simplest
method of tracking is to have the intelligence control the
motion of the missile so that the target remains within the
field. If this method of tracking is selected, restrictions
are immediately placed on the permissible motions of the

vehicle which must be considered by the aerodynamics specialist.
These restrictions depend not only on the aerodynamics of the
vehicle and the field of view of the intelligence device but
also on target contrast, characteristics of tracking circuits,
and behavior of the servo mechanism in the absence of homing
signals.

When a missile is to be released blind, the aerodynamics
specialist can compute trajectories for various release con-
ditions and so provide estimates of the time at which a target
in a specific location relative to the point of release will
lie within a specified field of view. The relation between
field of view, servo mechanism, and aerodynamical character-
istics must be such that tracking will be preserved. The
controls must be sufficiently effective to check any overshoot
or the servo mechanism must have a memory to bring the vehicle
and field of view of the intelligence device back on the target.
For some types of vehicle, the aerodynamic design can be made
such that the trajectory without any homing signal will include
the desired target within the field of view. The larger the
field of view, the easier the task.

Before release of the vehicle, the target must be brought
into its field of view, and tracking in range established.
During launching and thereafter the motion of the vehicle must
be such as to maintain the target within the field of view of
the intelligence device. Memory circuits within the intelligence

head may allow the signal to fade out for short time intervals and resume tracking in range when the signal level is restored. If during the period of no signal return, the vehicle removes the target from the field of view, the intelligence device is unable to secure further information of target direction. The permissible motion will depend upon the intensity of the signal returned by the target. If the target signal is only a small amount of the background signal, the effective cone of vision is reduced (in present radar homing equipment) to about 70 percent of its maximum width. Thus a smaller change in attitude will be required to lose directional tracking than if the target signal is much larger than the background. Since the average signal level and its consistency in amplitude depends on target size, target orientation, and on meteorological conditions, it is difficult to give definite design rules. However, the smaller the field of view, the smaller the change in attitude required to reduce a low signal to the background level, and from these considerations a large field of view is desirable.

The behavior of the servo mechanism in the absence of signal or more exactly just following fading of the signal has a definite bearing on the relation between field of view and permissible motions of the vehicle. If the servo mechanism maintains the vehicle on the course it was flying, the field of view could be small without risk of losing the target outside of this field should the signal fade for a few seconds. If

however the servo mechanism maintains the rate of turn and
pitch which exist at the time of signal fading, the target would
probably pass outside a small field of view before the signal
returned. Either of these types of performance of a servo
mechanism is somewhat idealized and not accurately obtainable
in any actual mechanism. The maintenance of the same course
is of advantage when the vehicle is nearly on the desired course
and loss of signal is due to fading.

When the vehicle is initially coming on course, it may
overshoot by a sufficient amount to lose track. If this occurs,
a servo mechanism which maintains the course of the vehicle
at the time the signal is lost will thereafter give no
opportunity for again picking up the target signal. The amount
of overshoot permissible will be larger, the larger the field
of view and the greater the target contrast.

Some of the restrictions on the motion of the vehicle which
are imposed by a narrow field of view can be removed by the
use of an intelligence device fitted with automatic directional
tracking. In this system the output of the intelligence device
is used to drive servo motors to center the field of view on
the target independent of the motion of the vehicle. The control
of the vehicle itself is then derived from the relative position
or rate of motion of the intelligence device with respect to
the vehicle, or both. The minimum permissible field of view
is then limited only by the precision and speed of response of

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the servomechanism. The extra degrees of freedom may give rise to more difficulty with stability of the two servomechanisms, one driving the intelligence, the other the vehicle. There has, as yet, been no field experience with a missile control of this type.

Background Signal.

All electronic intelligence devices have a certain internal noise level which can not be less than that produced by thermal agitation of electrons in the input circuit. The magnitude of this internal noise sets a lower limit to the signal which can be detected. However in actual practice there is a much higher background signal representing the signal return from areas other than that of the target which also lie within the field of view. Thus in a radar homing device the background signal is the reflected radar energy from the land, rough sea, or other obstructions that happen to be at the same range as the target. It may be very small or zero when the target is an airplane and the background is cloudless sky. In an optical homing device the background signal is the reflected or emitted optical energy from land, sea, or sky.

The important attribute of this unwanted background energy is its variability not only from place to place but at the same place, especially with weather. In the radar case against ship targets, the "sea return" depends very greatly on the height and shape of waves and on the orientation of the wave

troughs with respect to the receiver. The performance of a
reder homing missile against a specified target is affected very
much by the condition of the sea surface, the permissible range
at release being reduced as the sea becomes heavier because
signal returned by the ship is lower and the amplitude of signal
fluctuation is greater. Small tergets may be lost in the sea
return regardless of their range. Similarly, missiles using
other perts of the electromagnetic spectrum encounter background
signals which usually depend greatly on meteorological con-
ditions.

The ratio of target to background signal may show large
short-time varietions during the flight of a single missile,
and if the target contrast is not sufficient may produce
difficulties in tracking at long range.

Fluctuation of Signal Intensity.

As a homing missile approaches its target, the signal
intensity and usually the background signals also increase very
greatly, making necessary an automatic gain control in the
electronic equipment. The time constant of the gain control
must be short to take care of the rapid change at the end of
the flight but not so short as to obscure the variations of
scanning frequency which give the directional information. The
reliability of the information obtainable is dependent on the
target contrast which is a function both of the background
signal and the strength of the target signal itself. In

addition to the slower variation of signal strength as the
missile approaches the target, there may also be more rapid
variations associated with the changing geometrical relation-
ship between missile and target produced by the motion of the
missile and the linear and angular motions of the target. Such
fluctuations are always found in radar reflections.

The presence of fluctuating signal intensity and fluctuating
background signal means that the missile and servo mechanism
can not be designed on the assumption that information as to
the angular bearing of the target is continuously available.

The effects of fluctuating signal intensity which must be
guarded against are possible resonance effects in the servo-
mechanism, synchronization with the frequency of scanning, and
loss of tracking. It is not practical to lay down methods of
design. However, in the radar case the M.I.T. field experiment
group working on Pelican and Bat have found it advantageous to
make photographic records of variations of signal strength of
actual targets, to construct a special signal generator which
emits variable signals controlled by a cam cut to the observed
variation, and to test the effects of such a signal input on
the intelligence device output.

III. Maneuverability of Vehicle and other Aerodynamic Problems.
The Six Degrees of Freedom.

The trajectory of a missile is determined by the force of
gravity and the reactions between the missile and the air

through which it flies. The force of gravity acts vertically
downward through the center of gravity of the missile. It is
convenient to represent the resultant of the aerodynamic
reactions by three mutually perpendicular force components
acting through the center of gravity and three moments acting
about the three axes along which the force components are taken.
These forces and moments on a given missile are functions of
the shape of the missile, the orientation relative to the
direction of motion of its center of gravity, the spesd, the
axis and amount of angular rotation, and the density and other
physical properties of the air.

The missile has six degrees of freedom, three linear and
three angular. The interest of the user of a missile lies
essentielly in the three linear degrees of freedom, i.e. in
the linear motion of the center of gravity of the missile. The
angular motion of the missile is of interest only in so far
as it modifies the three force components and thus the trajectory.
A spinning or angular hunting motion is of no interest if the
missile strikes the target, a result dependent only on the
path of the center of gravity. It is generally true for the
ordinary bomb, propelled aircraft, or glider that an absence
of angular motion gives a more predictable and constant
trajectory. But in some other missiles, for example, shells,
a spinning motion is deliberately provided to provide a more
stable and predictable trajectory.

The dynamics of a missile, even if restricted to airplane-
like or bomb-like objects, is a very complex subject, and hardly
appropriate for this report. Readers are referred to Vol V of
W. F. Durand's Aerodynamic Theory for a discussion of airplane
dynamics by B. Melvill Jones. Only elementary and general
aspects will be discussed here.

The path of the missile can be controlled in a number of
ways, but the most usual method is through changes in the aero-
dynamic reaction by means of changes in shape of the missile.
The other methods may be advantageous in special cases. Thus
by the use of rocket motors it is possible to apply reaction
forces on the missile to change its path. This method is oper-
ative in the stratosphere where the air density is very small
and in free space. Control requires the ejection of a part of
the missile, a process differing only in degree from the burning
of fuel to produce power to operate other types of control
devices. Where other types of control are possible, it is
usually more economical to use them.

A missile may be controlled by varying its mass distribution
to modify the position of the center of gravity, thus changing
the resultant moments of the air reaction, hence the orientation
of the missile, which in turn modifies the force components.
This was done in early airplanes by motion of the pilot but the
method has been little used since that time.

It is possible to use power driven devices such as pro-

pellers or turbojets or to use thermal jets to produce forces
to modify the path or to use such devices to apply moments to
the missile to change its orientation and thus produce forces
to modify the path. It is more common to use these devices
as propulsion elements, and some control of the path, especially
changes in the vertical plane, is accomplished by varying the
propulsion force.

The most common method of control is through changes in
shape of the missile which usually alter the moments of the
air reaction and change the orientation of the missile in
addition to modifying the force components directly. The use
of a power driven propeller may be regarded as a special case
of a periodic change in shape. All changes of shape for
purposes of control involve the application of power, which
may be derived from any of the usual types of power sources.
It is obviously desirable that the power required for control
be small. This has led to the conventional type of control
used on aircraft in which the primary effect of the controls
is to apply moments to the missile which in a time determined
by the angular moments of inertia and the magnitude of the
applied moments changes the orientation of the missile to the
direction of motion of its center of gravity. As a result of
the change in orientation the forces are changed and the path
of the center of gravity is modified. We shall see later that
there are advantages in the design of homing aeromissiles in

selecting a change in shape which produces little or no change
in moment but does produce directly changes in the force com-
ponents. Such a method requires greater power for operating
the controls than the conventional method.

Equilibrium and Trim.

An unpropelled missile can be in complete equilibrium only
if the moments of the air reaction about three mutual per-
pendicular axes through its center of gravity are zero and if
the resultant air force is equal to the weight of the missile
and acts vertically upward. Such a state does not exclude the
possibility of a spiral or spinning motion. In fact the tail-
spin of an aircraft is a steady motion in which the above con-
ditions are fulfilled. We shall however not consider such
motions further, although there is no logical necessity of ex-
cluding spinning missiles. The guiding of such missiles would
seem to introduce many technical complications.

The equilibrium of a propelled missile differs only in
that the resultant aerodynamic force must equal the resultant
of the weight and the propelling force.

Practically all missiles now used or under consideration
have one or two planes of symmetry and a longitudinal axis
which lies within 10° of the intended direction of motion. The
exact location of the longitudinal axis is usually chosen to
suit the convenience of the specific problems but always in
a plane of symmetry. The other mutually perpendicular reference

axes are called the lateral and normal axes, and if the missile
has only one plane of symmetry, this plane contains the longi-
tudinal and normal axes. In the practical construction of air-
craft or missiles it is found impossible to make the device
sufficiently accurately to insure that, when flown or released,
the aerodynamic moments about the reference axes will be zero.
It is always necessary to apply control moments of suitable
magnitude about all three axes, or if it is desired to have the
controls in a given neutral position with no force applied to
the control levers, to provide adjustable trim tabs. These
adjustments are easily made when a human pilot is on board, but
other provision must be made when unmanned missiles are to be used.

For unbalanced moments about the lateral and normal axes,
a stable missile compensates by angular rotation to new positions
of equilibrium, since displacements about these axes produce
restoring moments. The missile would then fly at a somewhat
different angle of attack than planned and at an angle of yaw
which would give rise to a lateral force producing a lateral
drift of the missile. An unbalanced moment about the longitudinal
axis, which lies approximately in the direction of motion, can
not be compensated in this way, because rolling about the
direction of motion produces no static restoring moment. The
only methods known of compensating this unavoidable and undesired
moment arising from lack of symmetry in the actual unmanned
missiles involve automatic trimming by control surface dis-

placements under the control of one or more gyroscopes. The most
desirable method is to compensate the undesired moment directly
by displacement of ailerons. In some cases, for example the
German robot bomb V-1, the autopilot moves the rudder thus forcing
the bomb to travel at an angle of yaw sufficient to produce a
rolling moment due to yaw equal to the unbalanced rolling moment.
This method of correction gives rise to a lateral drift which
is one source of error contributing to the dispersion. One of
the results of the early work on the Pelican and Bat developments
was the demonstration that provision must be made in the auto-
pilot for compensating for undesired aerodynamic moments arising
from lack of symmetry, i.e. "trimming" the missile, and that a
gyro or equivalent reference is essential. It is fairly well
known that pendulums or aerodynamic surfaces whose position is
controlled by aerodynamic reactions are ineffective for this
purpose. In the Azon and Razon developments it has also been
found desirable to introduce automatic trim devices to eliminate
rolling motions, although not there required for stability
reasons, since these missiles have two planes of symmetry. The
elimination of the rolling motion in those missiles simplifies
the control problem as will be discussed in the section on
coupling between controls.

The state of complete equilibrium is rarely attained in
the practical use of missiles. The linear motion of the center
of gravity is an accelerated one, the mass times the acceleration

being equal to and in the direction of the resultant force.
The time required to reach equilibrium is often much greater than
the time of flight of the missile. For example, a falling
bomb is in complete equilibrium only when it reaches its
terminal velocity, a process requiring fall from a great height
and many tens of seconds. This long time constant arises from
the limited rate at which energy is supplied from the gravi-
tational field. In the case of an aircraft the slow phugoid
oscillation arising from interchange of kinetic energy and
potential energy has a period of approximately $0.22V$ seconds
when V is the speed in ft/sec, i.e. 88 seconds for a missile
traveling at 400 ft/sec. The design of servo mechanisms and
intelligence devices can not be based on the assumption of
equilibrium conditions.

Fortunately the time constants of the angular motions are
much shorter, usually of the order of a fraction of a second
or at the most a few seconds, increasing somewhat with the size
of missile but decreasing with its speed.

Magnitude of Lateral Forces, Angular Rates, and Radii of Curvature.

Consider a symmetrical missile falling vertically with its
longitudinal axis also vertical. The forces acting are the
force of gravity and the air resistance. Because of symmetry
there are no lateral forces. The downward acceleration will
be equal to the difference between the acceleration of gravity
and the ratio of the air resistance to the mass of the missile.

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The acceleration will ultimately approach zero as the missile approaches its terminal velocity at which the air resistance equals the weight. At any point along its length the trajectory may be modified only by introducing a lateral force. This force imparts a lateral acceleration which causes the missile to travel in a path which is approximately circular for some time. To move the missile in a path of radius R requires a centripetal acceleration of V^2/R where V is the velocity of the missile. The rate of change of direction of the trajectory $\frac{d\theta}{dt}$ equals V/R.

The usual method of securing a lateral force is to change the orientation of the missile so that its axis makes an angle to the trajectory. The missile does not then travel in the direction of its axis. The change in its direction of motion is dependent on the magnitude of the lateral force produced by the change in orientation in relation to the mass of the missile. If the force is small or the mass is large, the trajectory will be modified very slowly, even though the axis of the missile is at a large angle to the direction of motion of the center of gravity. The missile behaves in the same manner as an automobile traveling on ice when the steering wheel is suddenly turned.

Experience from tests on bombs and airfoils shows that the lateral force produced at a given angle of the missile to its trajectory is approximately proportional to the square of the speed and to the projected lateral area. A reasonable value

of the lateral force is about 10 lb/ft^2 on the fins or wings at a speed of 100 ft/sec and about 1/2 lb/ft^2 on a body of revolution at a speed of 100 ft/sec for angles of yaw of 15°, although with larger angles of yaw still higher values can be obtained. Extremely large angles of yaw give large drag forces which slow up the missile and thus reduce the lateral force. The lateral acceleration to be expected is therefore about $\frac{10A}{W}(\frac{V}{100})^2$ g where A is the area of the fins or wings in square feet, V the speed in ft/sec, W the weight of the missile in pounds, and g is the acceleration of gravity in ft/sec^2. In any actual design, the lateral acceleration should be determined from wind tunnel measurements on a model of the missile. A radius of curvature R requires an acceleration V^2/R. Hence

$$V^2/R = \frac{0.0322AV^2}{W}$$

or $\qquad R = 31 \frac{W}{A}.$

The rate of change of direction $\frac{d\theta}{dt} = \frac{AV}{31W}$.

For the standard 2000 lb bomb, A for the standard fins is about 4 square feet. By a suitable rudder, it may be expected that lateral forces of the above magnitude may be reached, in which case R = 15,500 feet and $\frac{d\theta}{dt} = \frac{V}{15500}$ rad/sec. = 0.0037 V degrees/sec. In the next 500 feet of fall after the rudder is applied, the bomb would move laterally about 7 1/2 feet.

It is to be noted that for this case of a vertical tra-
jectory, the radius of curvature obtainable with a given fin area
does not depend on the speed, since both the required and the
available acceleration vary as the square of the speed. However
the forces which the missile structure must withstand increase
proportional to V^2/R.

When the trajectory makes an angle to the vertical, the
force of gravity has a component at right angles to the tra-
jectory. It is customary to measure the angle from the horizontal
and to denote its value by θ. The gravity component is then
$g \cos \theta$. The path then curves downward unless a sufficiently
large lateral aerodynamic force overcomes the gravitational
component. Calling the lateral aerodynamic force L, we have the
following equation for the radius of curvature of the path

$$m\, V^2/R = mV\, \frac{d\theta}{dt} = mg \cos \theta - L$$

Consider first the case in which L is zero, i.e. a con-
ventional bomb. We find $R = \frac{V^2}{g \cos \theta}$ and $\frac{d\theta}{dt} = \frac{g \cos \theta}{V}$. The
maximum value of $\frac{d\theta}{dt}$ occurs when the axis of the bomb is horizontal
and equals $\frac{g}{V}$ rad/sec. At a speed of 320 ft/sec, $\frac{d\theta}{dt} = 0.1$ rad-sec =
5.7°/sec, and R = 3200 feet. As the speed increases, $\frac{d\theta}{dt}$ decreases
and R increases.

Next assume that the trajectory of the missile is to be
approximately a straight line to the target as is usually desired
for a homing missile. In this case the average value of L must

equal mg cos θ. The available control then depends on the changes in L which can be effected by the controls, and the control is the same as for a vertical trajectory. However, the requirement L = mg cos θ is the requirement for rectilinear flight and the speed required is dependent again on the area of fins or wings. The minimum speed for rectilinear flight, assuming the use of the maximum control, is given by

$$\left(\frac{V}{100}\right)^2 \frac{10Ag}{W} = g \cos \theta$$

$$\text{or} \qquad \frac{V}{100} = \sqrt{\frac{W}{A} \frac{\cos \theta}{10}}$$

For the 2000 lb bomb on a 45° trajectory, V is about 600 ft/sec.

At lower speeds than the minimum speed for rectilinear flight, the rate of change of direction is not as great and the radius of turn is larger. Values for any particular case can be estimated. The control obtainable depends greatly on the ratio of the speed to the rectilinear flying speed, and the rectilinear flying speed is determined mainly by the area of fins or wings.

Non-lifting Missiles.

From the aerodynamic point of view the properties of missiles in which the average value of the lateral force is zero are so different from those for which the average value is not zero that the two groups deserve separate treatment. Although an aircraft or glider could be trimmed to give zero lift, its general.

flight behavior would not be satisfactory, and the non-lifting
missile usually takes the form of an elongated object with two
planes of symmetry or more, and in some cases a body of revolution.
The simplest example is an ordinary bomb or rocket stabilized
by tail fins. When the axis is inclined, there is a restoring
moment because the line of action of the resultant force passes
behind the center of gravity. To keep the axis at an angle to
provide a lateral force, this moment must be balanced by a
smaller force in the opposite direction applied by a rudder at
the tail or a spoiler at the nose. The action of the rudder or
spoiler must be such that the missile still has sufficient
static and dynamic stability about the new position of equilibrium.

Missiles of this type are best adapted to steep trajectories
for the following reasons. Launching speeds are usually limited
to a few hundred feet per second. Control against the deflecting
action of gravity is effective only near the rectilinear flying
speed. The minimum rectilinear flying speed is of the order of
$100\sqrt{\dfrac{W}{A}\dfrac{\cos\theta}{10}}$. As the path approaches the vertical $\cos\theta$ de-
creases, the minimum rectilinear flying speed decreases, and
the control improves.

A decrease in $\dfrac{W}{A}$ also decreases the rectilinear flying speed
and hence improves the control at low speeds. This principle
has been used in the Roc project in which additional surface
has been provided to give larger lateral forces for a given angle
to the trajectory.

the motion of a non-lifting missile in the horizontal plane is of course independent of gravity and the general considerations discussed for a vertical trajectory apply.

When control moments are applied to missiles of the non-lifting type, about two axes at right angles to each other, the resultant air reaction can not be represented solely by a resultant force. There is also a resultant couple producing rotation about the longitudinal axis. In other words the missile has some of the characteristics of a screw propeller or wind mill. It has usually been found desirable to provide ailerons controlled by gyro elements to keep the missile from rolling. Such a control is desirable to permit easy separation of right-left from up-down controls of the missile.

It is inferred from the successful use of the German V-2 rocket, which is a non-lifting missile, that the problems of control of missiles of this type at supersonic speeds will not be too difficult.

As long as the missile follows the normal line of approach, it presents no difficulty when the controls are operated. If, however, the yaw or ...

entation) in a straight path which is inclined at an angle to
the vertical. If propelled, the path may be inclined upward.
Even if the controls are maintained in the neutral position, the
missile will finally approach an equilibrium state of un-
accelerated fall vertically downward at its terminal speed.
For an ordinary bomb or other missile of high wing loading the
time required corresponds to fall through a very great height.

The lifting missiles are intended to follow an approximately
straight flight path, which for powered missiles may be horizontal
or inclined upward. We shall consider first an unpowered
missile, i.e. a glider.

The only forces acting on a glider in flight are the force
of gravity and the resultant air force. It is customary to
consider the resultant air force in terms of its components
perpendicular and parallel to the direction of motion, the lift
and the drag. In equilibrium gliding flight the resultant of
lift L and drag D must balance the weight and hence must act
in the vertical direction. The flight path is therefore in-
clined downwards at an angle θ such that $\tan \theta = D/L$. The
resultant force R is usually expressed in terms of the dimension-
less coefficient C_R defined by $R = C_R A \, 1/2 \rho \, V^2$ where A is the
wing area, ρ the air density, and V the flight speed. Since
in equilibrium the resultant equals the weight W the equilibrium
speed of the glider along its flight path is determined by the
relation

$$V = \sqrt{\frac{W}{A} \cdot \frac{2}{\rho C_R}}$$

For an airplane the value of C_R increases from a small value
at 0° or some small negative angle of attack depending on the
shape of the wing section nearly linearly up to angles of the
order of 12 to 15°, reaches a maximum value of the order of 1.3
to 1.4 and than slowly decreases. The ratio of lift to drag
reaches a rather sharp maximum value at some angle between 5
and 10° and than decreases rapidly. The slowest equilibrium
gliding speed corresponds to the maximum value of C_R near the
stalling angle and in this region of angle of attack the glide
angle changes rather rapidly but with only small changes in
equilibrium speed. As the angle is reduced the glide angle
becomes flatter and the equilibrium speed increases. On
passing the angle of maximum L/D the glide path again becomes
steeper and the equilibrium speed increases still more. The
steepest path is the vertical dive in which the equilibrium
speed reaches its maximum value, the terminal speed at which
the weight is balanced by the drag.

The preceding description applies solely to steady state
conditions, i.e. those which occur after the lapse of a sufficient
interval of time. It is important to observe that an increase
in angle of attack at angles beyond that of maximum L/D at first
flattens the angle of glide or even causes a temporary escent
and the steeper glide angle is obtained only after the lapse of
sufficient time.

Let us suppose that while the airplane is gliding steadily

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at some angle of attack the angle is suddenly changed to e new
value. The lift at angles below the angle of ettack for maximum
L/D is changed much more than the drag and the principal effect
will be to produce an unbalanced force nearly perpendicular to
the direction of motion which will curve the flight path. Ultimately
of course the unbalanced drag force will modify the speed but
this process requires some time. Suppose the lift coefficient
corresponding to the steady straight flight at the instantaneous
altitude and speed is C_{L_e} and the actual lift coefficient is C_L.
The unbalanced force is then $(C_L - C_{L_e}) A \, 1/2 \rho V^2$ and hence the
lateral acceleration will be $\dfrac{C_L - C_{L_e}) A \, 1/2 \rho V^2}{W/g}$. The radius
of curvature of the path will be $\dfrac{W}{(C_L - C_{L_e}) Ag \, 1/2 \rho}$. This corres-
ponds to the value 31 W/A given earlier if $C_L - C_{L_e}$ is taken as
0.84. For a given wing loading and air density the radius of
curvature of the flight path depends on the difference between
the actual value of the lift coefficient and the value which
would be necessary in steady flight.

The minimum radius is obviously obtained with $C_{L_e} = 0$ and
C_L equal to the maximum lift coefficient, i.e. with a non-lifting
missile. The use of a lifting missile increases the minimum
radius of curvature and hence gives lower maneuverability.

For angles of attack beyond the angle of maximum L/D the
effect of the drag may predominate and the speed may be reduced
so rapidly that the lift force is not increased, although the
lift coefficient is greater.

The foregoing discussion applies to control of the path of the missile in a vertical plane. The lifting missile usually has larger surfaces, i.e. wings for support and control in the vertical plane. For right-left steering the lifting missile is usually designed to use the airplane method, i.e. banking or rolling the missile to obtain a component of the force on the large wing surfaces in the desired direction. The turns which can be produced without banking are of very large radius. An airplane with dihedral angle will automatically bank if the rudder is turned and ailerons are not moved, and will turn to right or left if the airplane is rolled either by deflecting the ailerons or by turning the rudder.

Turns of a glider are descending spirals. If the inclination of the spiral flight path to the horizontal is θ, and the radius of the spiral is r, the radius of curvature R of the flight path is $r/\cos\theta$. Call the angle of bank ϕ and the lift L. The available force is then $L\sin\phi$ and hence

$$L\sin\phi = \frac{W}{g}\,\frac{V^2}{R}$$

V being the flight speed, W the weight of the aircraft, and g the acceleration of gravity. Setting $L = C_L\,1/2\,\rho\,AV^2$ and solving for R

$$R = \frac{2}{\rho}\,\frac{W}{A}\,\frac{1}{C_L\,g\,\sin\phi}$$

The radius of turn can be decreased by increasing C_L as the glider is banked. This is the method commonly used in aircraft to make a tight turn. If C_L is zero, i.e. a non-lifting missile, banking

does not give rise to a turn, R being infinite. If C_L is not zero, i.e. a lifting missile, banking gives rise to a turn even if C_L is not increased by action of the longitudinal control. This is one of the essential differences between the aerodynamic properties of lifting and non-lifting missiles. We shall discuss further the right-left steering of lifting missiles by banking in the section on coupling of controls.

It is quite possible to design a lifting missile which could turn without banking. A large vertical surface would be needed and various practical difficulties arise.

Non-lifting missiles will probably have less difficulty with compressibility effects at high speeds. The chief difficulty with the lifting missile arises from the fact that the missile is unsymmetrical about the plane of the wings and the orientation is maintained by reactions on wings and tail whose moments about the center of gravity are equal and opposite. Compressibility effects first appear on the wings, usually producing diving moments and large changes in attitude. The non-lifting missile on the other hand is symmetrical and both body and fin moments are zero except when control is applied. Thus the trim position of the non-lifting missile is not likely to change with speed in the absence of control. The control will undoubtedly be modified by compressibility effects.

At the present time aerodynamic data at supersonic speeds are quite limited, consisting mainly of information on the drag

of projectiles obtained in firing tests. Adequate supersonic
facilities are now being provided and within the next year or
two a great deal of the necessary basic research should be
completed.

Powered Missiles.

The control of powered non-lifting missiles requires little
further discussion. When no control is applied, the propulsive
force acts in the direction of motion. Variation of the pro-
pulsive force changes the speed but not the direction of motion.
When control is applied, the orientation may change, in which
case the propulsive force has a component at right angles to
the direction of motion of the center of gravity, increasing the
lateral force available for control. This effect is usually
not large.

In the case of powered lifting missiles, variation of the
propulsive force constitutes a second and independent method
of control in the vertical plane. Under equilibrium conditions
the speed is approximately independent of the value of the
propulsive force, satisfying the equation

$$V = \frac{2W \cos \theta}{A \rho C_L}$$

For horizontal flight the propulsive thrust T is equal to the
drag D. If T is greater than D the missile climbs at angle θ
such that $T = D + W \sin \theta$. The angle θ may be negative corres-
ponding to descent if T is less than D. For more complete
discussion reference should be made to one of the many books

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on the subject of airplane performance.

Homing Missiles.

In homing missiles, the apparatus contained within the missile locates the target with reference to some axis fixed in the missile. The information so obtained is not adequate unless this reference axis is always in a known relation to the flight path of the center of gravity of the missile. The simplest solution is obviously to make the reference axis coincide with the flight path, i.e. to have the intelligence device "look" along the flight path. The installation is especially simple if the application of control does not change the orientation of the missile with respect to its flight path. The securing of this result is one of the aerodynamic problems peculiar to homing missiles. The solution used in the Pelican and Bat projects was to change the lift of the wing by trailing edge flaps and to so locate the center of gravity of the missile and a fixed tail structure that the downwash effects on the tail produced moments counterbalancing the moments produced by the flap deflection. The Roc project adopted the same solution.

In the flight of an unpowered homing missile in still air against a stationary target, the flight path is approximately rectilinear. The initial speed is usually much less than the terminal speed in the straight glide path and hence the speed increases. To maintain the rectilinear flight, the lift must be maintained constant and equal to $W \cos \theta$. As the speed increases the lift coefficient must be decreased by the action

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of the controls.

It appears desirable to operate unpowered homing missiles
within the range of lift coefficients lying between zero and
that for maximum lift-drag ratio. As is well known the equi-
librium flight path first flattens and then steepens as the lift
coefficient is increased. There is accordingly a reversal of
control as regards the final effect when past the maximum lift-
drag ratio. Missiles are not usually in equilibrium on their
flight path, and computation shows that the first effect of
control is always that of the change in lift. If however the
lift-drag ratio at the maximum lift coefficient permitted by
the controls corresponds to a slope of path steeper than the
actual path slope to the target, the control proceeds to its
limit and stays there. The intelligence calls for a higher lift.
A higher lift coefficient is obtained as the control moves
toward its limit but also a higher drag coefficient. The drag
reduces the speed so that the actual lift does not increase very
fast and the flight path corrects very slowly, if at all. It
is probably desirable that powered homing missiles should also
be operated in the region between zero and maximum lift-drag
ratio.

If the homing missile is powered, various combinations of
control are possible. For example, the propulsive thrust might
be controlled by a speed-sensitive device to maintain a constant
speed in the later stages of the flight.

Stability Problems.

In theory a homing missile might receive sufficient control information to give stable flight without inherent stability of the missile in the absence of control. In practice, a missile must have satisfectory stability to maintain its flight in periods of intentional or unintentional absence of control information.

It is not practical to review the many aspects of the stability of missiles. The disturbed motions of a stable airplane-like missile in the absence of homing control consist of various combinations of a rapid heavily damped longitudinal motion in which the angular pitching motion is most prominent; a slow oscillation, the previously mantioned phugoid oscillation, in which the missile rises and falls with the spaed decreasing and increasing; a rapidly damped rolling and yawing oscillation; and e slow spiral motion.

Under certain conditions the missile may pass from steady rectilinear flight to a steady spin. In a true spin, as distinguished from the spiral motion referred to above, the wing surface is stelled, i.a. meets the air at a large angla. The spinning motion is howevar a steady motion with the following balence of moments and forces;

1. The stalled wing rotates of itself at such a speed that the rolling moment is zero. (A wing at normal angles offere resistance to rolling; e stalled wing is in unstable equilibrium at zero rate of roll).

2. The aerodynamic pitching moment tending to reduce the angle of attack is balanced by the centrifugal pitching moment tending to increase the angle of attack.

3. The aerodynamic yawing moment is balanced by the centrifugal yawing moment. The spinning characteristics are greatly affected by the angle of yaw at which this balance occurs.

4. The airplane descends at such a rate that the vertical component of the air force equals the weight.

5. The horizontal component of the air force gives the requisite centripetal acceleration of the center of gravity towards the spin axis.

A missile can be made difficult to spin by a suitable disposition of tail surfaces to give large aerodynamic pitching moments and a favorable anti-spin equilibrium angle of yaw.

When a homing device is applied to an airplane like missile, the disturbed motions take on a somewhat different character. For example, considering only the longitudinal stability, the slow phugoid oscillation disappears and is replaced by a damping of any disturbance of the velocity along the flight path, the period and damping of the fast angular pitching oscillation are controlled in part by the static longitudinal stability of the missile and in part by the power of the control surface and the lag of the servomechanism, and there arises a second angular

pitching oscillation of longer period which may in some cases
degenerate into two aperiodic motions. As the missile approaches
the target, the frequencies and damping constants change some-
what and an additional mode appears.

In a similar manner the slow spiral motion associated with
the lateral stability of the free flying missile disappears.
There are short and long period combined rolling and yawing
oscillations where damping may be positive, zero, or negative
and the long period motion may degenerate into aperiodic motions.

Some aspects of the stability of homing missiles will be
discussed briefly in the section on System Stability or Hunting
Problems.

IV. Coupling Between Controls

Interdependence of Controls.

A missile in flight has six degrees of freedom but if it
follows conventional aircraft design it has only three controls
unless it is propelled in which case it has also a throttle or
equivalent thrust control. The three controls are usually
movable surfaces to control the moments about three mutually
perpendicular axes and there is no direct control of the linear
motions. Unfortunately the three controls do not give independent
effects. Thus the rudder produces a small rolling moment as
well as a yawing moment which turns the aircraft to right or
left. When yawed, there results a much larger indirect rolling
moment arising from the aerodynamic reactions. The ailerons

produce yawing moments as well as rolling moments and the yawing
moments may be "adverse" i.e. the resultant yaw produces a
rolling moment in a direction opposite to the desired rolling
moment or they may be favorable.   In extreme cases the ailerons
may turn the aircraft to right or left without rolling it or
the rudder may roll the aircraft with little yaw.   The effects
may change sign for the same aircraft at different speeds i.e.
at different angles of attack or at different lift coefficients.
The interaction between the rolling or yawing motion and the
pitching motion is fortunately very small.

In the case of non-lifting missiles, operation of the left-
right and up-down controls simultaneously produces rolling
moments and hence it has been found that a non-lifting missile
must have ailerons if rolling motions are to be avoided.

Correspondence between Intelligence Coordinates and Controls.

The intelligence devices generally available for use in
homing aeromissiles give information on the bearing of the
target to the right or left and up or down from some reference
axis, i.e. an essentially two-dimensional presentation.   In
the case of radar devices, range information can also be obtained
if desired.   The missile however has three controls if un-
propelled and at least four if propelled.   There is evidently
a problem in making connection between the two-dimensional out-
put of the intelligence device and the control system of the
missile.

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In the case of a non-lifting missile, the obvious method
of connecting the controls is to connect the two channels of
the intelligence device to two control surfaces producing
moments about two mutually perpendicular lateral axes which are
aligned with the intelligence device. While the two control
surfaces may be designed to give independent action about the
lateral axes, the application of control about both axes
simultaneously will produce rolling moments about the longi-
tudinal axis. If the aerodynamic design were such that the
longitudinal axis remained in the direction of motion, the roll
would not be objectionable unless the rate was so high that
the lag in the control system introduced phase errors. However
most missiles of the zero lift type change orientation as the
controls are applied. The roll takes place about the axis
of the missile and introduces incorrect error signals in the
intelligence device. The better solution is to stabilize the
missile in roll by means of ailerons controlled from a suitable
gyro system. Thus in the simplest method, the roll control is
governed by a gyro, and the other two by the channels of the
intelligence device.

There have been many ingenious suggestions for connecting
the controls of a non-lifting missile to permit continuous
rotation of the missile. It does not seem profitable to discuss
them here. For remotely controlled as distinguished from homing
missiles such devices become computers, often of complex design,

for transforming from fixed to rotating axes, and metering the
degree of control to be given by each of the two control surfaces.

In the case of a lifting missile of the airplane type,
turns are accomplished by banking the airplane and it is not
therefore desired to prevent roll. It is necessary either to
reduce the number of control surfaces from three to two or to
couple two of the control surfaces together to be controlled
from a single intelligence channel. If the missile is powered,
the throttle or equivalent power plant control may be arranged
to be controlled by airspeed, altitude, or some quantity
associated with engine performance as for example mixture ratio,
peak pressure, temperature, etc. Combinations may be used
but it has not been customary to use the throttle for up-down
control of the flight path.

The more common two-control airplanes use the elevator-
rudder or the elevator-aileron combinations. Both methods have
been used in the design of missiles. With proper design reason-
ably satisfactory turns can be made either with ailerons or
with rudder. For homing missiles, aileron control is believed
to give somewhat smaller errors, since out-of-trim rolling
moments can be balanced without introducing yaw. In either case,
the use of two controls alone makes possible independent
connections of the two intelligence channels.

It has already been pointed out that a gyro is required
for trimming the missile in roll. It is desirable that this

gyro also limit the maximum angle of roll; otherwise the missile is likely to turn over on its back when a control signal is continued for some time.

In automatic pilots for aircraft, the rudder and aileron ere often controlled together for turns. Such a coordination is possible for one or at most a narrow range of flight conditions. For missiles intended to operate over a considerable speed range, the rudder displacement for a given aileron displacement varies, both with airspeed and angle of attack, and it has not seemed necessary to attempt the design of the necessarily complicated control. By suitable aerodynamic design of a two-control missile, the side-slip during turns can be made reasonably small.

Effect of Roll on Error Signals.

The lifting missile of the airplane type banks during a turn. The reference axes of the intelligence device are fixed with reference to the missile and hence rotate with the missile. The axis of rotation of the missile does not usually coincide with the direction of motion of the center of gravity of the missile. Hence the intelligence device no longer measures the up-down and right-left errors with respect to axes fixed with respect to the vertical.

It has been previously pointed out that an airplane is made to turn in its tightest circle by banking and then pulling back on the stick to increase the lift coefficient to its

maximum value. The coupling between the up-down and right-left
controls produced by banking automatically gives an error signal
to the elevator or elevons in a direction to increase the lift
coefficient, and thus accelerates the turn. At the same time
the increased lift bends the trajectory of the center of gravity
upward. The exact behavior of the system depends on the
characteristics of intelligence device, servo-mechanism, and
missile.

The error angles are readily computed for idealized motions,
If the missile rotated about the direction of motion of the
center of gravity, the effects amount simply to rotation of the
axes of reference of the error signals. If the error angles
referred to the original axes are δ_e in elevation and δ_a in
azimuth, and if referred to axes rotated through an angle ϕ
δ_e' and δ_a', we have the relations

$$\delta_a' = \delta_a \cos\phi - \delta_e \sin\phi$$
$$\delta_e' = \delta_a \sin\phi + \delta_e \cos\phi$$

If however the rotation occurs about an axis making an angle δ_{e_o}
with the direction of motion of the center of gravity

$$\delta_a' = \delta_a \cos\phi - (\delta_e - \delta_{e_o}) \sin\phi$$
$$\delta_e' = \delta_a \sin\phi + (\delta_e - \delta_{e_o}) \cos\phi$$

In the general case the dynamics of a rigid body with six
degrees of freedom and the aerodynamic characteristics of the
control must be considered. Thus the axis of torque varies
with the displacement of the control surfaces. The body rotates

not about the axis of torque, unless this axis is also a principal
axis of inertia, but about an axis intermediate between the
axis of torque and a principal axis. Thus the instantaneous
axis of rotation travels in the body and an integration process
is necessary. Because of these complications it is not very
practical to correct for these effects by inserting a computing
device between the intelligence and the controls. One of the
important consequences of the foregoing effect is that when
there is an elevation error angle and the true azimuth angle
is zero, an azimuth error is passed on to the controls equal to
$-(\delta_\epsilon - \delta_{\epsilon_0})$ sin ϕ. To fix ideas suppose that the target is high
so that $\delta_\epsilon - \delta_{\epsilon_0}$ is positive and that the airplane rolls as for
a right turn (positive ϕ). A negative azimuth error is then
given to the controls, which tends to oppose the right turn.
If however $\delta_\epsilon - \delta_{\epsilon_0}$ were negative, i.e. true elevation error zero
but axis of roll above flight path or target is low, the azimuth
error given to the controls would assist the turn. In the first
case there would be a damping effect on oscillations; in the
second a destabilizing effect which would promote hunting. In
the usual lifting missile the effect is on the average a de-
stabilizing one.

Effect of Angle of Attack or Yaw on Error Signals.

In the non-lifting missile, the application of controls
pitches or yaws the longitudinal axis so that the intelligence
no longer looks along the flight path. Using the same notation

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as before, the effect is given by the relation

$$\zeta_a' = \zeta_e - \zeta_{e_o}$$

The same relation applies to the elevation control of a lifting missile. It is in theory possible to interconnect the controls with the intelligence device to move the reference axes of the intelligence device to compensate for this effect. If undercompensated there will be a stabilizing action against oscillations and if overcompensated a destabilizing action.

Methods of Reducing Coupling Between Controls.

The methods of reducing both the interdependence of the controls and the undesired effects of the angular motions on the error signals are still in the early development stages and much additional aerodyn... research is required. A homing missile ca ... made to work with all these effects present in some degree. It would seem fruitful however to attempt to make the normal flight path coincide with the longitudinal principal axis of inertia, for all control positions, and to do further research on reducing interactions between controls.

V. System Stability or Hunting Problems.

Illustration of Hunting.

Perhaps the most difficult of the special problems associated with the design of homing missiles is that of securing a satisfactory stability of the complete system, since the overall stability depends on the characteristics of missile, intelligence, and servomechanism and especially on their interrelations. To

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illustrate the problem there are shown in Figs. 2, 3, and 4 the observed motions of three homing missiles. In Fig. 2 it is seen that the missile rolls and yaws in increasing oscillations of about 10 seconds period until the missile strikes the ground. In extreme cases such a missile may turn completely over on its back, lose track of the target and descend in a steep spiral. In Fig. 3, the missile pitches in a steady oscillation of nearly constant amplitude and from 5 to 7 seconds period. Figure 4 shows a rolling and yawing oscillation which is damped as the flight progresses. The period of the main oscillation is approximately 10 seconds but there is superposed a rapid roll oscillation of about 4° amplitude and 1 1/4 second period.

The steady hunting motions illustrated in Figs. 3 and 4 are objectionable only in so far as they affect the path of the center of gravity of the missile. The unstable motion of Fig. 2 usually but not always results in complete failure of the missile and a wide miss.

The effect of a given hunting motion on the trajectory may be estimated as follows. The error angle is equal to the ratio of the velocity dy/dt of the missile transverse to the flight path to the velocity V of the missile, provided the missile looks along its flight path. If this angle oscillates sinusoidally with maximum amplitude θ_0, the motion may be represented by

$$\frac{dy}{dt} = V\theta_0 \cos \frac{2\pi t}{T}$$

where T is the period. The lateral displacement y is then given
by the formula

$$y = y_0 + \frac{V\theta_0 T}{2\pi} \sin \frac{2\pi t}{T}$$

where y_0 is the mean value of y. The maximum excursion from
the trajectory is then $\frac{V\theta_0 T}{2\pi}$. For V = 400 ft/sec, θ_0 = 3° =
1/20 radian, T = 2π seconds, the flight path oscillates approxi-
mately 20 feet. On the other hand if T = 1 second, the flight
path oscillation is only 3 feet.

These simple computations underestimate the error since
the actual missile does not look accurately along the flight
path under dynamic conditions. This effect is however greatest
for the short period oscillations where the errors are small.

Factors Leading to Instability.

The presence of error signals from a homing device would
seem at first sight to remove most of the sources of instability
and to supplement the aerodynamic damping forces which are
present in any normal design. The element which introduces
difficulty is the unavoidable lag between the presence of an
error angle and the application of the corrective forces and
moments to the missile. The lag may arise in the mechanical
and electrical parts of the intelligence device, control system,
or servomotor or may be introduced by the motion of the missile
in response to the controls in a manner to introduce errors
in the measurement of the error angle by the intelligence device.

The physical effect of the lag in producing hunting may

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be appreciated by considering the missile displaced from the correct flight path toward the target but being returned to it by the action of the controls. As the error angle approaches zero, the control surface deflection is approaching zero and should reach zero at the same instant as the flight path intersects the target. Or if the control is of the on-off type, reversal should occur at the instant the flight path intersects the target. If this could occur, the aerodynamic damping forces would absorb some of the kinetic energy present and the next crossings would occur with successively decreasing speed until the missile reached equilibrium on a steady trajectory. Because of the inevitable lag present, the control surface remains deflected in a direction to increase the error angle of the missile for a short time and thus feeds energy into the oscillatory motion. Equilibrium is reached at an amplitude of oscillation such that the amount of energy fed in is equal to that absorbed by the aerodynamic damping. It may happen that the equality of energy fed in and absorbed does not occur for any amplitude, in which case the oscillation builds up until the missile capsizes.

The amount of energy fed in is directly proportional to the time lag, and to the overall sensitivity i.e. to the magnitude of the correcting forces and moments produced by a given error angle. This overall sensitivity is dependent on such factors as the area of the control surfaces, the speed, altitude, and

attitude of the missile, the deflection or rate of deflection
of the control surface for a given electrical input to the
servomechanism, and the sensitivity of the intelligence device.

The aerodynamic damping is a function of altitude and
airspeed as well as of relative areas of various parts of the
missile and of the amplitude of oscillation.

The time lag in the intelligence device is usually determined
by the amount of integration or smoothing found desirable to
give a reasonably steady output signal from the intelligence
device. For the radar homing systems used so far, the time
lag has been within the range 0.05 to 0.25 seconds.

The time lag of the .... al electromechanical servo system
is of the order of 0.10 to 0.15 seconds, but considerably smaller
values can be obtained by special design.

Either a time lag or a time advance may be introduced by
the relations between the motions of the missile, the axis of
the intelligence device, and the direction of motion of the
center of gravity of the missile as discussed in the section
on the effect of roll on the error signals. For the usual
design of missile the axis of roll lies between the longitudinal
axis of the missile and the direction of motion of its center
of gravity and a time lag is introduced. Thus if a disturbance
rolls the missile say for a right turn, a false azimuth error
is indicated in a direction to aid the turn and amplify the
effect of the disturbance. Or from the point of view of lag,

if the missile is off course to the left and correcting to the
right, the centering or reversal of control is delayed until
the missile moves off course to the right a sufficient amount to
bring the rotated reference exis on the target.

The use of automatic lead computers to reduce errors
associated with wind and moving targets often leads to a difficult
stability problem as discussed later in this report.

Anti-hunt Devices.

Since hunting is evidence of a phase lag between the error
angle and the application of corrective forces and moments, the
remedy is obviously the introduction by some means of a phase
advance which is greater than the lag. As the missile comes on
course, the control must be reduced or removed before the error
is actually zero. The most common method of accomplishing this
result is to incorporate a rate term in the control, i.e. to
make the position or speed of the control dependent in part on
the error and in part on the rate of change of the error.

A compensating or anti-hunt circuit can often be introduced
in the amplifier or output circuit of the intelligence device
itself. It usually takes the form of a suitable condenser-
resistance network which may be considered either as a phase
advancing device for sinusoidal signals or as introducing a rate
component for arbitrary signal variation. Difficulties often
arise with such a circuit if the input signals are "rough", i.e.
fluctuating in magnitude for a given error angle as for example

is the case for radar reflections from many types of targets.
Because of the antihunt circuit, the higher frequency "roughness"
is amplified much more than the slower variation as the missile
comes on course. A further characteristic of this anti-hunt
circuit is that good performance is obtained only with consider-
able attenuation of the input signal so that additional amplification
is required.

The rate term may be introduced by a gyroscopic turn
indicator which measures the angular rate at which the missile
comes on course. This method has been found completely effective
in obtaining system stability. It has the disadvantage that
winds or moving targets require a steady rate of turn which
introduces a control signal. The presence of this signal in
effect limits the maximum rate of turn and thus increases the
errors associated with winds and moving targets.

Time to Come on Course.

One of the characteristics of the control system of interest
is the time required for a missile released with an initial error
angle to come on its proper course. The time can be controlled
to some extent by the overall sensitivity of the control system.
It has already been pointed out that increasing the overall
sensitivity increases the amount of energy fed into the system
when a time lag is present. Even if the system is stable, the
missile may come on course with a damped oscillatory motion and
the damping is small if the sensitivity is too great. A return

without oscillation can be secured with suitable values of the
control parameters. Both experience and theory show that it is
necessary to make a compromise between the stability character-
istics and the time to come on course. In most cases it is
necessary to make the time to come on course a matter of the
order of 10 seconds or more.

Effect of Wind Gusts.

Wind gusts introduce disturbances which excite the natural
modes of motion of the complete system. The effects persist for
times which depend on the periods or time constants of the
natural modes, and the disturbed motion at any instant depends
on the history of disturbances over comparable past periods of
time. Excluding the effects of changes in speed along the tra-
jectory for which the time constant is very large but which in
themselves do not cause the missile to miss the target, there
usually exist modes for which the time constant is of the order
of 10 seconds. Hence gusts in the latter part of the flight
path introduce errors which can not be wholly corrected in the
time available..

Experience has shown decreased dispersion in tests of homing
missiles over water as compared with tests over land which is
probably to be attributed to the decreased gustiness over water.
In one instance over land, a severe disturbance was noted from
the sharp boundary between a heated layer of air at the ground
and a cold air mass aloft. While the designer of a missile can

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vary the frequencies of the natural modes of motion through small
limits, it is practically impossible to change their order of
magnitude. Fortunately the errors in the trajectory decrease
as the rectilinear flight speed increases.

Methods of Analysis.

The problem of the design of stable systems has been
approached by several methods of mathematical analysis and by
experiments on mechanical, electromechanical, or electrical
models. If sufficient information is available from tests of
the component parts, the choice of control parameters to assure
system stability can be made by any of these methods of analysis.
The purely mathematical methods require more or less idealized
representation of the performance of the component parts of the
system and are most useful for systems whose performance is
described by linear differential equations. The various methods
using models have the advantage that some of the actual com-
ponents can be incorporated as part of the model, so that non-
linear control mechanisms and on-off or step controls whose
mathematical analysis is often difficult can readily be inves-
tigated.

The control system of a homing missile constitutes a
closed-cycle control system or servomechanism. The chief differ-
ence from ordinary servomechanisms is that the parameters of
the system are not constant but vary through considerable limits
during the flight of a single missile. More specifically the

parameters assoc.uted with aerodynamic forces and moments are
functions of the air speed, air density, and attitude of the
missile and its control surfaces. If these variations are taken
into account, the usual methods of analysis of servomechanisms
may be applied. These involve a study of the response of the
system to certain standardized conditions, the two most useful
conditions translated in terms of a homing missile being a sinu-
soidal displacement of the target at various frequencies or a
sudden permanent displacement of the target. If the system is
a linear one, its performance is completely known when the per-
formance under either of the above conditions is known.

Readers who are interested in the details of the mathematical
procedures are referred to the restricted paper by A. C. Hall
of the Servomechanisms Laboratory, Massachusetts Institute of
Technology, entitled "The Analysis and Synthesis of Linear
Servomechanisms", Technology Press, 1943. This paper describes
the application of the sinusoidal analysis to the design of
linear servomechanisms with continuous control. An example of
analysis using suddenly applied disturbances may be found in
Restricted Technical Note No. 809 of the National Advisory
Committee for Aeronautics by F. H. Imlay entitled "The Theoretical
Lateral Motions of an Automatically Controlled Airplane Subjected
to a Yawing Moment Disturbance". Robert T. Jones of the same
laboratory has developed this method of study of the control
of certain types of missiles.

The preceding methods of analysis lend themselves to a
complete study of the performance of the system, including not
only the question of whether the system is stable or not but
ell questions of the magnitude of the errors. For determination
of stebility, one may apply the method of small oscillations
commonly used in airplane stability problems. This method as
epplied to an airplene without automatic pilot is described in
great detail by B. Melvill Jones, in Division N, Vol 5 of
Durand's Aerodynamic Theory under the title "Dynamics of the
Aeroplane". The method in effect determines the period and
damping of the naturel modes of oscilletion and the damping of
the neturel aperiodic motions. If the damping constent turne
out to be negative, the corresponding mode is unsteble.

The mathematical methode are in practice limited to syetems
described by linear differential equations and find some
difficulty in the treetment of actual servo systems with friction,
"dead" regions, and time lag. For this reeson models have been
found useful. These range in complexity from systems repre-
senting e single degree of freedom of the missile to more complete
flight tables which include the three enguler components of the
motion of the "phantasmagoria" which simulate one or two
components of the linear motion. As an example, methods of
automatic roll stabilization may be investigated with the actual
gyro and servo elements by using a mechanical system (damped
pendulum or damped rotor) to simulate the inertie and damping

of the missile about its roll axis. A system of this type as
well as a more complicated electro-mechanical model of the
longitudinal motion were used in the study of the Pelican and
Bat control systems. In addition, Dr. A. C. Hall of the Servo-
mechanisms Laboratory, Massachusetts Institute of Technology,
developed a flight table in which the pitching, yawing, and
rolling motions of the missile with proper cross coupling were
fully simulated and on which an overall test could be made of
the complete control system including intelligence unit, gyro-
scopes, and servomechanism. This flight table was extremely
useful in determining the best adjustments of the several.
parameters of the control system.

Experience with these model methods and comparison with
actual flights of homing missiles show that the models reproduce
the angular motions encountered in flight extremely well.
Designers of homing missiles will find that the use of this
method of studying system stability and overall performance will
save much time in the development of a stable system and in
adjusting it for best performance.

VI. <u>Problems arising from Wind and Moving Targets.</u>
<u>Pursuit Curves</u>

If the target is moving or there is a natural wind, a flying
missile which always heads toward the target follows a path
known as a pursuit curve or homing curve depending on whether
the motion is referred to the air or to the ground. The pursuit

curve has the characteristic that the radius of curvature becomes
very small as the target is approached and becomes infinitely
small for the idealized case of a point target. The maneuver-
ability of the missile is limited and hence the missile will not
hit a moving point target or a stationary target in a cross
wind. The magnitude of the miss depends on the speeds of the
missile, wind, and target and their relative directions, on the
range, and on the maneuverability of the missile as expressed
by its minimum radius of curvature.

This problem in idealized form has been studied by the
Statistical Research Group, Division of War Research, Columbia
University, under the direction of the Applied Mathematics Panel.
For convenience the effect of wind and target motion are combined
to give an apparent target motion of speed v. (See Figure 5). The
azimuth ϕ_0 of the launching position referred to the direction
of apparent motion of the target, the launching radius r_0, the
ratio n of the apparent target speed v, to the speed of the
missile V, and the minimum turning radius ρ_m of the missile are
the quantities determining the miss for the case of point target.
Constant velocities of target and missile, direction of launching
toward the target, and missile continuing along an osculating
circle when the minimum radius of curvature is reached are
assumed.

The computed miss M is shown in Figure 6 in the form of
a plot of M/ρ_m vs ϕ_0 for $n = 0.1$, 0.2, 0.3 and 0.4 and $r_0 = 6\rho_m$.

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$10\rho_m$, and $14\rho_m$. The essential points to notice are (1) that the maximum miss varies but little with launching radius; (2) that the maximum miss is approximately equal to $1/2\rho_m n^2$ and (3) that the miss becomes very small for ϕ_o between 140° and 180°. The only factors within the control of the designer of the missile are the speed of the missile and its minimum turning radius. The speed ratio should be as small as possible and the minimum turning radius as small as possible to minimize pursuit curve errors. The user of the missile may reduce the error by making his attack on a course such that ϕ_o is approximately 145°. In the case of ship targets this course gives a good compromise between the magnitude of the error and the projected length of the ship which determines the permissible error which still yields a hit.

Computers.

The essential feature of the pursuit curve is that the missile moves along a path which makes an angle $\tan^{-1} \dfrac{n \sin \phi_o}{1+n \cos\phi_o}$ with the line joining the missile and target. If the axis of the intelligence device were rotated with respect to the longitudinal axis of the missile by this amount in the opposite direction to the apparent target motion, the path with respect to axes fixed in the target would be straight and the missile would travel on a "collision" course. If the user of the missile has the necessary data to compute n and ϕ_o, and the means of offsetting the axis of the intelligence device by the computed

amount and if the variations of n and ϕ_o may be neglected, the pursuit curve error may be eliminated.

It is in theory possible to design a computer which automatically offsets the axis of the intelligence device by the required amount. The simplest scheme in theory is to equip the intelligence device with a separate servo system which always keeps the axis of the device centered on the target. If the missile flies a collision course the bearing of the target will remain constant and hence the angular velocity of the intelligence device will be zero. If the controls are so arranged that the missile is guided to make the angular velocity zero, the course followed should be the desired collision course. In practice a high precision is required since the rate of change of bearing for a small offset is very small at long ranges. Furthermore the missile is subject to numerous disturbances of attitude and it appears that an accurate linear integrating angular velocity meter would be required. It is not known whether the scheme would work or not; it has not yet been tried on a missile.

Another possibility which amounts to integrating the angular velocity is to determine the angular displacement of the axis of the missile after a certain time or distance by means of a free gyro which maintains a fixed direction in space. A knowledge of the angular displacement and the range at the beginning and end of the displacement permits a computation of the desired offset. This computation also assumes a constant value of n

throughout the flight.

Still another possibility is a continuously integrating angular velocity meter which continuously sets in the appropriate offset.

Study shows that the roughest sort of correction reduces the pursuit curve errors even though a true collision course is not obtained. Hence it is worthwhile to experiment with the simplest conceivable computers, for example those dependent merely on the relative frequency and length of error signals in the two opposite directions.

The chief bar to the general use of computers lies in their effect on system stability and in fact the automatic devices described can not be used because they produce excessive hunting. The computer essentially puts in a lead on a moving target. If the missile as a result of a disturbance rotates to increase the lead, the angular velocity developed acts to increase the lead still further and thus builds up an oscillation of increasing amplitude. The computer can not distinguish between angular velocity arising from disturbances and angular velocity caused by the missile following a pursuit curve. An automatic lead computer is an inherently destabilizing device.

The two methods of solving the stability problem are (1) to feed in only a fraction of the correct lead angle or (2) to introduce a time constant in the integration process which is long compared to the natural periods of oscillation of the

homing missile. In the first case the permissible fraction is
dependent on the stability reserve of the complete system of
intelligence device, servocontrol, and missile. In the second
case the time constant must be of the order of at least 20 or
30 seconds for missiles exhibiting natural periods of the order
of 1 to 8 seconds.

The computer problem is susceptible of mathematical analysis
for any specific missile and control system, and is not too
difficult if all elements of the system are linear. Actual
experience with computers on a homing missile is not yet available.

VII. Bore Sight Errors.

If the axis of the intelligence device does not coincide
with the direction of motion of the center of gravity of the
missile, there arises an error which may be termed the bore sight
error. Let us confine our attention to the case where the
intelligence device is not equipped with a computer or other
means of introducing an offset angle, i.e. the axis is fixed
with respect to the missile. The missile then flies at a fixed
angle to the line joining missile and target and hence follows
a logarithmic spiral whose equation in polar coordinates is
$\rho = \rho_o e^{-\frac{\theta - \theta_o}{\tan \epsilon}}$ where ρ_o and θ_o are the coordinates of the release
point and ϵ is the bore sight error. As the missile approaches
the target the radius of curvature decreases and finally becomes
equal to and would be less than that which the missile can follow.
The radius of curvature of the path is equal to $\rho/\tan \epsilon$ or if ϵ

is small to ρ/ϵ. Thus if ρ_m is the minimum radius of turn of the missile, this value is reached at a value of ρ equal to $\rho_m \epsilon$. This value of the range to the target at which the radius of turn becomes ρ_m will be designated R_m. If it is assumed that the missile continues to travel in a circular path of radius ρ_m, the miss is readily computed to be $1/2\rho_m\epsilon^2$. For if the path were straight the miss would be $R_m\epsilon = \rho_m\epsilon^2$ and the circular path produces a correction $1/2\rho_m\epsilon^2$ leaving a residual miss of $1/2\rho_m\epsilon^2$. For $\rho_m = 10,000$ feet, $\epsilon = 1°$, $R_m = 167$ feet and the miss is only 1.4 feet. The miss increases as the square of the bore sight error.

Bore sight errors arise from many sources in addition to the obvious one of inaccuracy of construction of the vehicle and of the mounting brackets of the intelligence device. Both mechanical and electrical imperfections of the intelligence device may give rise to bore sight error. Thus in a radar homing device the electrical axis of the antenna system may not coincide with the geometrical axis because of unsymmetrical distribution of dielectric or conducting material near the antenna. The output circuits may be unbalanced such that the output is not zero when the target is on the axis of the antenna. The servo control system may contain elements which are not balanced when the input signal is zero; for example, the pick-off of a rate gyroscope may be displaced from the correct zero position. Errors from these and similar causes may be controlled by careful inspection tests of the individual components and an overall check of an

installed system can readily be made. The reference axis of
the vehicle can be oriented with respect to the target until
the indicated output is zero in which case the angular dis-
placement of the target from the reference axis gives the bore
sight error.

The most difficult problem is to determine that reference
axis of the vehicle which lies in the direction of motion of
the center of gravity of the vehicle in free flight. In fact
a vehicle designed like a conventional airplane travels at
different attitudes for different positions of the control sur-
faces. Particularly in the vertical plane the angle of attack
varies with the elevator setting. Thus there is a variable
bore sight error which might amount to as much as 10° or more.

A similar error may occur in the horizontal plane when
rudder and elevator control are used. For in this case the
rolling moment due to any lack of symmetry of the missile about
its longitudinal axis must be balanced by application of rudder
to yaw the missile which then flies at an angle to its longi-
tudinal axis.

Two methods have been used to reduce bore sight errors from
aerodynamic causes. The first and most satisfactory is to
design the missile so that the change in attitude with application
of control is as small as possible. This requires aileron-
elevator control rather than rudder-elevator control, so far
as the horizontal motions are concerned. In the vertical plane

the aerodynamic design should be such that the lateral forces
are modified without change in attitude. One method is to use
flaps on the trailing edges of the wings to change the lift and
to balance the pitching moments so produced by pitching moments
from the tail arising from changes in the downwash angle. This
can be done by proper choice of tail size and location and of
center of gravity location of the missile.

The second method which has been used to reduce boresight
errors from angle of attack changes is to mount the intelligence
device on trunnions and couple it to the controls in such a
manner that the reference axis of the intelligence device is
rotated to compensate for changes in angle of attack. This com-
pensation can be made perfectly for steady flight conditions
but there are residual errors under dynamic conditions. In
addition a stability problem arises as in the case of computers
and usually only a partial compensation can be made if hunting
troubles are to be avoided. In some instances such a coupling
of intelligence device to controls has been used as a means of
damping as previously discussed. In such a case the bore sight
error is not completely eliminated.

A final source of bore sight error is the inevitable in-
accuracy of construction of the vehicle itself as regards its
external shape. The axis of zero moment i.e. the direction of
travel will vary somewhat from one missile to another of the
same intended shape. Such errors can be measured in a suitable

wind tunnel but only with great expenditure of time and the re-
quirements for uniformity of flow in the wind tunnel are difficult
to meet.   In the case of Pelican and Bat missiles, errors of
about 0.8° were found from this source.

   With sufficient care and neglecting actual equipment failures
it should be possible to keep the bore sight error to the order
of magnitude of 1°.

VIII. Effects of Sensitivity, Lag and Minimum Range of
       Intelligence Device on Pursuit Curve and Bore Sight Errors

   The preceding discussion of pursuit curves and bore sight
errors has assumed certain ideal characteristics of the intelligence
device.   The facts that the intelligence device must look away
from the target to produce error signals and that it has a certain
lag have been neglected.

   Let us consider first an intelligence device without lag
which gives an output signal proportional to the error angle up
to a certain angle σ_m and which gives a constant output for
angles greater than σ_m .   The maximum output signal gives the
minimum radius of curvature of the path ρ_m .   For any error
angle σ smaller than σ_m it will be assumed that the radius of
curvature ρ_c is given by $\rho_c /\rho_m = \frac{\sigma_m}{\sigma}$; i.e. the radius of curvature
is infinite for $\sigma = 0$.   If then the discussion of the pursuit
curve is reexamined, it is seen that the axis of the intelligence
device must point away from the target by an angle σ when the
radius of curvature is ρ_c .

   An exact solution of this problem is difficult and has not

been carried out. We have already seen that the pursuit curve
error is entirely analogous to a bore sight error of magnitude
$\tan^{-1} \frac{n \sin \phi_o}{1+n \cos \phi_o}$ where n is the ratio of the apparent target
speed to the speed of the missile and ϕ_o is the azimuth of the
missile referred to the direction of apparent motion of the target.
This equivelent bore sight error varies during the flight of
the missile along a pursuit curve because ϕ_o varies and the
complication of pursuit curve calculations is the determination
of ϕ_o as a function of the initial azimuth, initial range, and
n. It will be instructive however to consider the simpler
problems of the effect of the value of σ_m on the miss arising
from a constant bore sight error ϵ .

This problem has been worked out by Dr. Skramstad(unpublished)
for the case where ϵ is small, the lateral displacement y of
the missile from a line joining the release point to the final
target position is small compared to the initial range $x_0 - x$,
and the slope of the trajectory $\frac{dy}{dx}$ to the same line is small
dompared to 1. The differential equation is found to be

$$\frac{1}{\rho} = \frac{d^2y}{dx^2} = \frac{\sigma}{\rho_m \sigma_m} = \frac{1}{\rho_m \sigma_m}\left[\left(\frac{y}{x-x_0}\right) - \frac{dy}{dx} + \epsilon\right]$$

The solution of this differential equation was found to be

$$y = \epsilon(x_0-x)\left[\log \frac{x_0}{x_0-x} \cdot \frac{H(x_0/\rho_m \sigma_m)}{x_0/\rho_m \sigma_m}\right] + \epsilon$$

$$\frac{dy}{dx} = \epsilon - \epsilon\log\left[\frac{x_0}{x_0-x} - \frac{H(x_0/\rho_m \sigma_m)}{x_0/\rho_m \sigma_m}\right]$$

$$\frac{d^2y}{dx^2} = \frac{-\epsilon}{x_0-x}$$

where $H(x) = 1 + \frac{1}{x} + \frac{2}{x^2} + \frac{6}{x^3} + \ldots$

For practical values of x_0, ρ_m, and σ_m the second term in the brackets is entirely negligible. If at a range $x_0 - x = R_m$ the missile is assumed to follow a circle of radius ρ_m the miss M is given by

$$M = y + R_m \frac{dy}{dx} - 1/2 \frac{R_m^2}{\rho_m}$$

Whence

$$M = \epsilon (R_m + \rho_m \sigma_m) - 1/2 \frac{R_m^2}{\rho_m}$$

The range R_m at which $\sigma = \sigma_m$, or $\rho = \rho_m$ is equal to $\rho_m \epsilon$

since $\frac{1}{\rho} = \frac{d^2y}{dx^2} = -\frac{\epsilon}{x_0 - x} = \frac{-\epsilon}{R}$

Hence the miss is given by

$$M = \rho_m \sigma_m \epsilon + 1/2 \rho_m \epsilon^2$$

If we assume $\sigma_m = \frac{1}{15}$ we obtain the following values:

ϵ	$\frac{M}{\rho_m}$
.05	.0046
.10	.0117
.15	.0213
.20	.0333
.25	.0480

The effect of the value of σ_m is zero if $\epsilon = 0$. Thus if the missile travels a collision course and the bore sight error is zero, there is no error from limited sensitivity of the intelligence device. When ϵ is not zero, the effect of σ_m is

to give an error term proportional to ϵ whereas the miss due to bore sight error alone is proportional to ϵ^{ν}. Hence the effect of σ_m predominates at small values of ϵ .

It is possible to use the formulae previously given to compute the pursuit curve errors by means of an equivalent bore-sight error. For the azimuth angle ϕ_o of the missile referred to the direction of the apparent of the target changes from ϕ_o to $\phi = \phi_o - \frac{dy}{dx}$. Introducing the value of $\frac{dy}{dx}$ in the expression for equivalent bore sight error, there is obtained

$$\epsilon = \frac{\sin\left(\phi_o - \epsilon + \epsilon\log x_0/\rho_m + \epsilon\log 1/\epsilon\right)}{n + \cos\left(\phi_o - \epsilon + \epsilon\log \frac{x_0}{\rho_m} + \epsilon\log 1/\epsilon\right)}$$

This transcendental equation can be solved for ϵ as a function of ϕ_o , n and x_0/ρ_m and with the expression for M in terms of ρ_m, σ_m , and ϵ the values of $\frac{M}{\rho_m}$ as a function of σ_m , ϕ_o , n, and x_0/ρ_m can be found. The accuracy is not very good when $|\phi - \phi_o|$ exceeds about 0.3.

Since large pursuit curve errors correspond to large values of ϵ , the term in σ_m is the smaller but for $\sigma_m = \frac{1}{15}$ corresponds to increasing the miss by 50 to 100 percent.

The effect of a time lag τ in the intelligence device may be estimated in a qualitative way by considering this effect also as an equivalent bore sight error. The error signal actually present at a range R is that appropriate to a range $R + V\tau$ where, V is the speed of the missile. This may be regarded as due to an error angle at range R differing from that actually present

intelligence device, $\rho_m = 10,000$ feet, $\epsilon_m = \frac{1}{15}$ and $\epsilon = \frac{1}{30}$, the miss is 89 feet. If R_m is reduced to 200 feet the miss is 29 feet of which 7 feet is associated with the minimum range and 22 feet with the radar sensitivity ϵ_m.

IX. Strength Problems of Homing Missiles.

The structural design of a missile is in most respects entirely analagous to that of an aircraft. It is desired here only to call attention to two design conditions peculiar to missiles.

For rocket or jet propelled missiles it is often inefficient to design the power plant of sufficient size to include the take-off condition. It is better to use assisted take-off by means of a catapult or by special launching rockets. In German experience with anti-aircraft and long range missiles, accelerations from 2 to 16g have been used in launching. The missile structure must be designed to withstand the inertia loads produced, and so must all of the parts of the intelligence device and control system.

The second design condition peculiar to missiles is that of full control application at maximum speed. Because of the bore sight error which may be present, the intelligence device will call for the maximum rate of correction. The missile must be able to withstand the loads so produced or some automatic device must be used to limit the acceleration which may be imposed.

X. Launching Problems of Homing Missiles.

Some discussion has been given of launching problems in the discussion of the relation between the field of view and the permissible motions of the missile. The target must come within the field of view and remain there. A sighting device or computer may be required for determination of the proper release time.

A special problem encountered in the release of missiles from aircraft is that of avoiding maneuvers which cause contact between missile and aircraft after release. It appears essential to keep the homing device disconnected from the controls until the missile is at a safe distance from the aircraft. The choice of air speed and attitude at release can be made after the study of computed trajectories confirmed by experimental tests. In general the air speed must be lower than the horizontal flight speed of the missile, the amount by which it should be lower depending somewhat on the drag-weight ratio of the missile. In some cases interference effects between the missile and the aircraft may cause an unfavorable trajectory. However no such effects have been encountered for missiles of high wing loading.

XI. Conclusion.

A survey has been given of some aspects of the design of homing missiles for flight through air, primarily to place on record that part of the experience in the study of the radar

homing glide bombs, Pelican and Bat, which is likely to be of value in the future development of homing missiles. None of the topics have been treated in detail in this paper, but it is believed that the discussion is sufficient to indicate the nature of the problems and possible methods of their solution.

Fig. 1 Directional sensitivity of antenna of Pelican radar receiver.

Fig. 2 Unstable oscillation of homing missile.

Fig. 3 Undamped oscilletion of homing missile.

Fig. 4 Damped oscillation of homing missile.

Fig. 5 Diagram of launching variables determining pursuit
    curve errors.

        r_0 distance to target at launching

        ϕ_o azimuth of launching position referred to
          direction of apparent motion of target.

        v apparent speed of target, resultant of wind
          effect and terget motion.

Fig. 6 Pursuit curve errors due to effects of wind, target motion,
    and limited maneuverability of missile.

        r_0 distance to target et launching.

        ϕ_o azimuth of launching position referred to
          direction of apparent motion of target.

        v apparent speed of target, resultant of wind
          effect end target motion.

        V speed of missile.

        n speed ratio v/V.

        ρ_m minimum radius of curvature which missile
          can be made to fly.
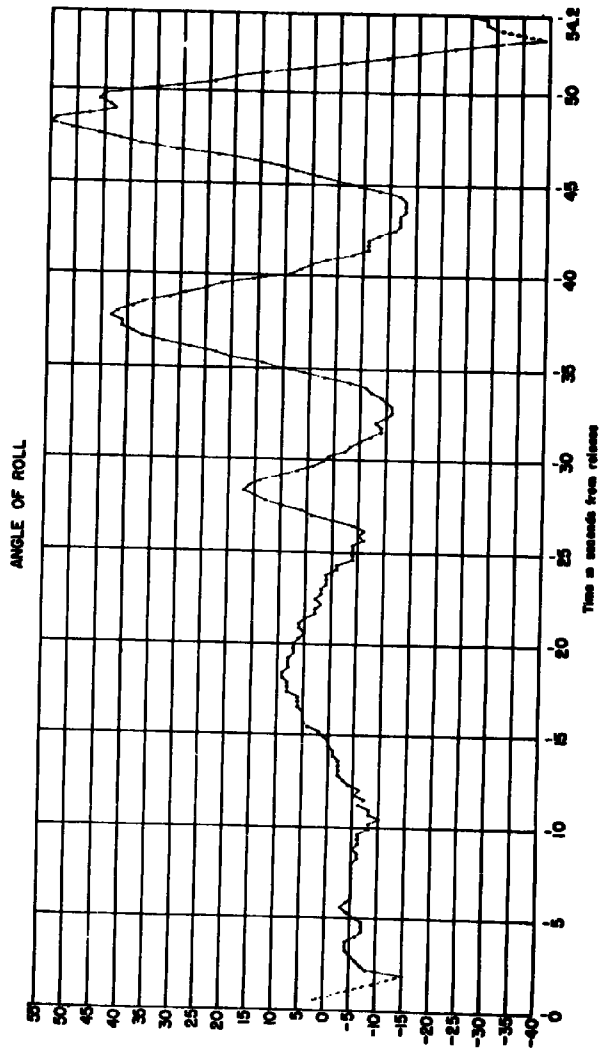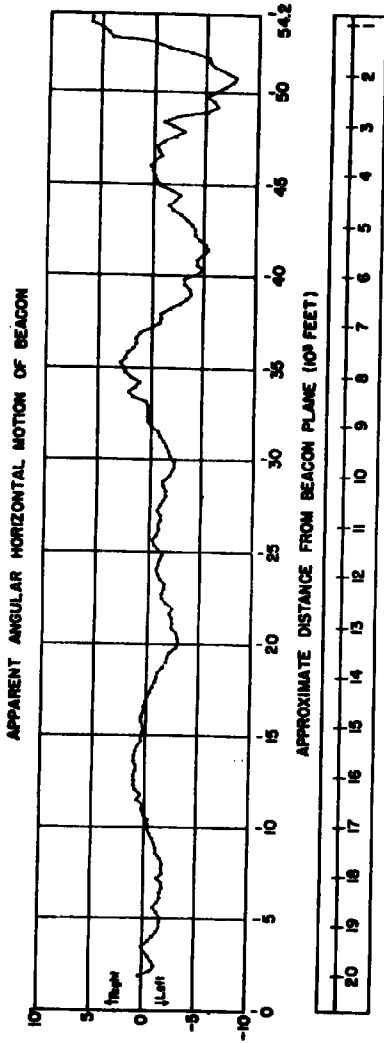
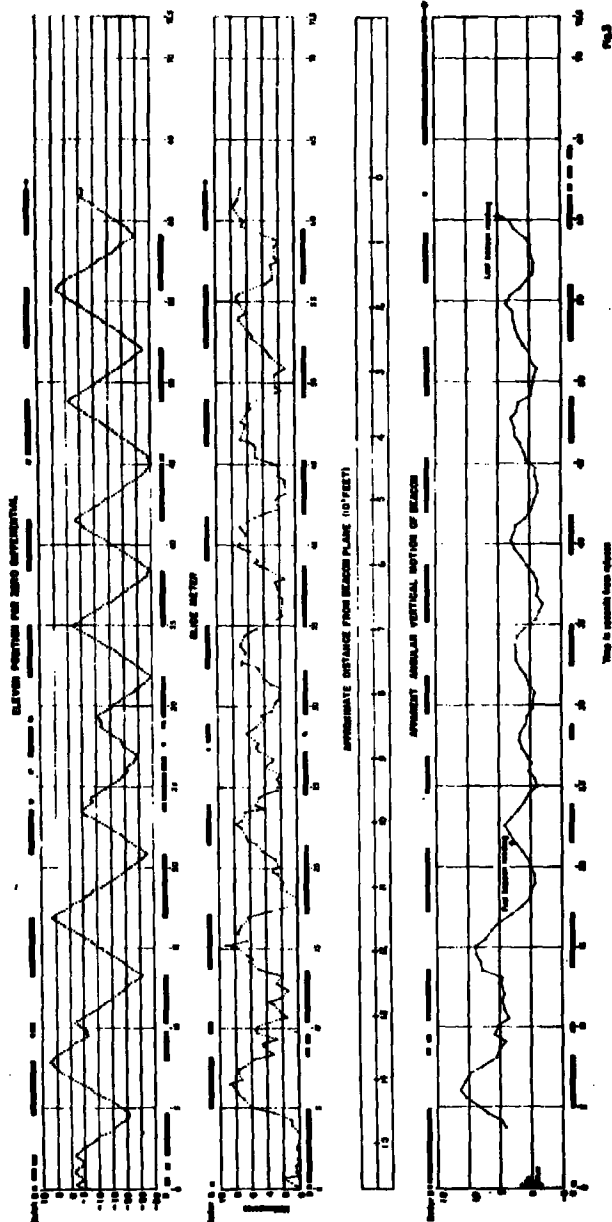   M miss referred to point target.

Fig. 1

APPARENT ANGULAR HORIZONTAL MOTION OF BEACON

APPROXIMATE DISTANCE FROM BEACON PLANE (10³ FEET)

ANGLE OF ROLL

Time in seconds from release

Fig.2

- 80 -

APPARENT ANGULAR HORIZONTAL MOTION OF BEACON

APPROXIMATE DISTANCE FROM BEACON PLANE (10⁴ FEET)

ANGLE OF ROLL

Fig. 5

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TITLE: Some Aspects of the Design of Homing Aero-Missiles

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ABSTRACT:

The report pertains to homing devices for guided missiles. There are five classes of missiles, and they are placed in each class according to source of control. The five classes are listed and their means of control are discussed. Maneuverability of guided missiles and aerodynamic problems pertaining to them are discussed in detail. In conclusion, the various problems pertaining to launching are given.

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At the request of ███████████, we have conducted a Mandatory Declassification

Review of the attached documents under the provisions of Executive Order 13526, section 3.5,

for public release. We have declassified the documents in full. We have attached a copy of our

response to the requester on the attached Compact Disc (CD). If you have any questions, contact

me by e-mail at storer.robert@whs.mil, robert.storer@whs.smil.mil, or robert.storer@osdj.ic.gov

or by phone at 571-372-0483.

Robert Storer
Chief, Records and Declassification Division

Attachment:
CD

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MAR   7   2013

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1. MDR request
2. Document 1